THE DAMPING OF PLASMA OSCILLATIONS IN A MAGNETIC FIELD

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The theory of plasma oscillations points up the unique role played by resonance particles (those which satisfy the condition $\omega_k - \mathbf{kv} = n\omega_H$; n = 0, 1, 2, ..., where ω_k and k are the frequency and wave vector of the oscillation, v is the particle velocity, and $\omega_H = eH / mc$), which, by exchanging energy with the waves, strengthen or weaken them. The important role of these particles in the damping of plasma waves is readily apparent from the fact that γ, the attenuation constant of the waves, in a rarefied plasma is proportional to the derivative of the distribution function of the resonance particles (see, for example, [1]).

According to the linear theory, an infinitely small perturbation which is created in the plasma will gradually damp out, and the system will return to the thermodynamic equilibrium state within a time of order $1/\gamma$. The quasi-linear theory (see [1]) indicates the existence of isolated states which can be attained by an unstable plasma as the result of a perturbation growing in it. These states are characterized by the fact that the distribution function f is constant in some regions of phase space (a "plateau" appears in the function f). This latter corresponds, according to the preceding comments, to the cessation of the oscillation damping. Such a state could be observed in a collisionless plasma in the absence of a magnetic field. Collisions between the particles tend to destroy the "plateau." When the collisions are taken into account, the distribution function approaches the Maxwellian form, and stationary absorption of the oscillations is established.

In the present work we investigate the influence of a weak magnetic field on the damping condition of Langmuir electron oscillations. It is found that the magnetic field prevents the formation of the "plateau" in the distribution function, so that stationary absorption can be established in the absence of collisions.

Thus the action of a magnetic field is in some sense similar to the effect of including collisions. If in the latter case the damping constant depends on the collision frequency, it is determined in the present case by the Larmor frequency of the electrons.

Let us assume that at the time t = 0, a one-dimensional spectrum of Langmuir electron oscillations is excited in the plasma. The oscillations are created in the interval $(k_0, k_0 + \Delta k_0)$ of wave vector space, where $\Delta k_0 \ll k_0$. We shall assume that the wavelength is much smaller than the Larmor radius of the electrons in the resonance region:

$$1/k \ll \omega_0 / k\omega_H, \qquad k_0 < k < k_0 + \Delta k_0. \tag{1}$$

Here ω_H is the Larmor electron frequency, and ω_0 is the plasma frequency.

We shall examine the problem of oscillation damping in the quasi-linear approximation. If the direction of the wave vectors ($k \parallel Ox$) is perpendicular to the direction of the magnetic field ($H \parallel Oz$), then the quasi-linear equation for the averaged distribution function will have the simplest possible form:

$$\frac{\partial f}{\partial t} = D(t)\frac{\partial^2 f}{\partial v_x^2} - \omega_H\left(v_y\frac{\partial f}{\partial v_x} - v_x\frac{\partial f}{\partial v_y}\right),$$

$$D(t) = \frac{e^2}{2m^2}\frac{|E^2 k_0(t)|}{v_0}, \quad v_0 = \frac{\omega_0}{k}. \tag{2}$$

Here $E^2 k_0$ is the spectral density of the oscillation energy, e is the electron charge, m is the electron mass, and v_i is the i-th component of the electron velocity.

In the following we shall confine ourselves to the case of a sufficiently narrow packet of oscillations (that is, in the region where the quasi-linear theory is applicable):

$$\frac{\Delta v v_0}{v_T^2} \ll 1, \qquad \Delta v_0 = \frac{\omega_0 \Delta k_0}{k_0(k_0 + \Delta k_0)} \tag{3}$$

and we shall assume that the characteristic diffusion time is much smaller than the time of electron passage through the resonance region, in that part of velocity space where the maximum absorption of oscillations takes place:

$$\frac{\Delta v_0^2}{D} \ll \frac{1}{\omega_H}\frac{\Delta v_0}{|v_y|}, \qquad \sqrt{3\Delta v_0 v_0} \lesssim |v_y| \lesssim v_T$$

or

$$\alpha = \frac{\omega_H \Delta v_0 v_T}{D} \ll 1. \tag{4}$$

In the inequality (4) we can set $D \approx D(0)$, since we are interested only in those oscillations whose amplitudes vary insignificantly during a time of order $\omega_H^{-1}(v_0^{-1})^{1/2}\Delta v_0$.

When the conditions (3) and (4) are satisfied, as will be shown below, only the derivative of the distribution function by v_x is strongly distorted in the resonance region, while the variation of the distribution function itself can be neglected. Accordingly, a quasi-stationary state is established in the resonance region during the time required for an electron to pass through it. Thus the problem is reduced to the solution of the stationary equation

$$D_0\frac{\partial^2 f}{\partial v_x^2} = \omega_H\left(v_y\frac{\partial f}{\partial v_x} - v_x\frac{\partial f}{\partial v_y}\right) \tag{5}$$

with the boundary conditions

$$D_0\frac{\partial f}{\partial v_x}\Big|_{v_x=v_0} = -\omega_H v_y[\varphi_0 - f|_{v_x=v_0}],$$

$$\frac{\partial f}{\partial v_x}\Big|_{v_x=v_0+\Delta v_0} = 0, \quad (v_y > 0),$$

$$\frac{\partial f}{\partial v_x}\Big|_{v_x=v_0} = 0,$$

$$D_0\frac{\partial f}{\partial v_x}\Big|_{v_x=v_0+\Delta v_0} = -\Delta_H v_y[\varphi_0 - f|_{v_x=v_0=\Delta v_0}] \quad (v_y < 0),$$

$$\varphi_0 = \frac{n}{2\pi v_T^2}\exp\left(-\frac{v_x^2+v_y^2}{2v_T^2}\right)$$

Here φ_0 is the unperturbed velocity distribution function of the electrons, and n is the density. We shall seek a solution of Eq. (5) in series form:

$$f = f_0 + \frac{\omega_H}{D_0}f_1 + \left(\frac{\omega_H}{D_0}\right)^2 f_2 + \ldots. \tag{6}$$

Using the boundary conditions on Eq. (5) and neglecting terms of second order in small quantities, we have

$$f = \frac{1}{1+\Delta v_0 v_0/v_T^2}\Big\{\varphi_0(v_0) + \frac{\omega_H}{D_0}\Big[\Big(v_x\frac{(v_0+\Delta v_0)^2}{2} - \frac{v_x^3}{6}\Big)\frac{\partial\varphi_0}{\partial v_y} + C(v_y)\Big]\Big\} \quad (v_y > 0),$$

$$f = \frac{1}{1-\Delta v_0 v_0/v_T^2}\Big\{\varphi_0(v_0+\Delta v_0) + \frac{\omega_H}{D_0}\Big[\Big(v_x\frac{v_0^2}{2} - \frac{v_x^2}{6}\Big)\frac{\partial\varphi_0}{\partial v_y} + c(v_y)\Big]\Big\} \quad (v_y < 0). \tag{7}$$

(The additive constant $c(v_y)$ can be calculated from the second approximation. For that purpose, it is found that an inequality stronger than (4) is required to make perturbation theory applicable; however, inequalities (3) and (4) are quite sufficient to make Eq. (8) valid. This can be verified by subjecting Eq. (2) to a Laplace transformation on v_y.)

We note that the zeroth approximation to the distribution function differs, in the resonance region, from the average value of the distribution function in the absence of a magnetic field. This is not surprising, because the foregoing values were calculated with the use of the same limit transitions (t tending to infinity and ω_H to zero) but in the reverse order.

Ignoring terms of order $v_0 v_T^{-2} \Delta v_0$, we can write the expression for the derivative of the distribution function by v_x as follows:

$$\frac{\partial f}{\partial v_x} = -\frac{\omega_H v_y (v_0 + \Delta v_0 - v_x) v_0}{D_0 v_T^2} \varphi_0 \quad (v_y > 0),$$

$$\frac{\partial f}{\partial v_x} = -\frac{\omega_H v_y (v_0 - v_x) v_0}{D_0 v_T^2} \varphi_0 \quad (v_y < 0). \tag{8}$$

Averaging over v_y, we find the damping constant

$$\gamma \approx \frac{\omega_H v_T \Delta v_0}{2D_0} \gamma_0 = \frac{4\pi nkT}{\varepsilon} \left(\frac{\Delta k_0}{k_0}\right)^2 \frac{\omega_H}{v_T k_0} \gamma_0 . \tag{9}$$

Here γ_0 is the constant according to the linear theory with no magnetic field taken into account, ε is the energy density of the oscillations, and nkT is the kinetic energy density of the plasma.

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REFERENCE

1. A. A. Vedenov, Problems of Plasma Theory [in Russian], Gosatomizdat, 3rd edition, 1963.

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